Analogy of speed and time of vehicles moving towards each other when the vehicle's individual lengths are negligible compared to the distance covered by them:

Case a): When 2 vehicles start at different time and different speed from points A and B and move towards each other:

When vehicle V_1heading towards point B from A start at X A.M and vehicle V_2 heading towards A from B start at (X+a) A.M where a is in hours,

and when speed of vehicles V_1 and V_2 are respectively S_1 and S_2 km/hr,

time taken by V_1 to reach the meeting point C = t hours and

time taken by V_2 to reach the meeting point C= (t-a) hours.

If d denotes the distance between A and B in km such that AC= (d-m) km and CB= m km,

$$(S_1*t) + (S_2*(t-a)) = d \text{……..} (1)$$

$$S_1*t = d-m \text{……..} (2)$$

$$S_2*(t-a) = m \text{…….....} (3)$$

If t_1 is the time taken in hours by V_1 to reach the remaining distance and t_2 is the time taken in hours by V_2 to reach the remaining distance,

$$t_2*S_2 = d-m \text{……...} (4)$$

$$t_1*S_1 = m \text{……...} (5)$$

Comparing equations (2) and (4),

$$S_1*t = t_2*S_2 \text{……..} (6)$$

Comparing equations (3) and (5),

$$S_2*(t-a) = t_1*S_1 \text{……} (7)$$

From equation (6),

$$S_1/S_2 = t_2/t \text{……} (8)$$

From equation (7),

$$S_1/S_2 = (t-a)/t_1 \text{……..} (9)$$

Comparing equations (8) and (9),

$$t_2/t = (t-a)/t_1$$

$$t_1*t_2 = t^2-(a*t) \text{……} (10)$$

Multiplying L.H.S of equations (8) and (9) and equating it with its R.H.S,

$S_1^2/S_2^2 = (t_2/t_1)*((t-a)/t)$……. (11)

$S_1/S_2 = \sqrt{(t_2/t_1)*((t-a)/t)}$…… (12)

Example: When a car1 starting from point A at 08.00 A.M heading towards point B with a speed of 60 km/hr and a car2 starting from point B at 11.00 A.M heading towards point A with a speed of 90 km/hr meets each other at 01.00 P.M, what are the time taken by car1 and car2 to cover their respective remaining distances?

Solution: As car1 and car2 start from their respective points with a time gap of 3 hours,

a = 3.They meet each other after 5 hours from the commencement of journey of car1 and

Hence t = 5 hours.

$t_1*t_2 = 5^2-(3*5) = 10$…… (From equation 10)

$t_1 = (10/t_2)$…….. (13)

$60/90 = \sqrt{((t_2/ (10/t_2))*((5-3)/5))}$……. (From equations 12 and 13)

Solving, we get,

$t_2 = (10/3)$ hours and $t_1 = 3$ hours.

Hence car1 and car 2 take 3 and (10/3) hours respectively to cover their remaining distances after their meeting point.

Case b): When 2 vehicles start at same time and different speed from points A and B and move towards each other:

In this case, the time difference ‘a’ becomes zero.

Hence equation (10) becomes,

$t_1*t_2 = t^2$…….. (14)

Equation (12) becomes,

$S_1/S_2 = \sqrt{(t_2/t_1)}$…….. (15)

Example:When a bike1 starting from point A heading towards point B travelling at a speed of 40 km/hr and bike2 starting from point B heading towards point A travelling at a speed of 70 km/hr meet each other after 6 hours, what are the time taken by bike 1 and 2 to cover their remaining distances if they start their journey simultaneously?

Solution: Here, $t_1*t_2 = 36$…… (From equation 14)

$t_1 = (36/t_2)$……. (16)

$40/70 = \sqrt{(t_2/(36/t_2))}$…….. (From equations 15 and 16).

Solving, we get,

$t_1 = (21/2)$ hours and $t_2 = (24/7)$ hours

Hence bike1 and bike 2 take (21/2) and (24/7) hours respectively to cover their remaining distances after their meeting point.

Case c): When 2 vehicles start at different time with same speed from points A and B and move towards each other:

In this case, $S_1 = S_2$.

Hence equation (1) becomes,

$(S_1*t) + (S_1*(t-a)) = d$

$S_1*((2*t)-a) = d$……. (17)

Till the meeting point,

$S_1*t = d-m$…….. (18)

$S_1*(t-a) = m$…….. (19)

After the meeting point,

$t_1*S_1 = m$……... (20)

$t_2*S_1 = d-m$……. (21)

Comparing equations (19) and (20),

$t = t_1+a$…….. (22)

Comparing equations (18) and (21),

$t_2 = t$…… (23)

Example : When a moped1 starting from point A at 07.00 A.M heading towards point B and a moped 2 starting from point B at 11.00 A.M heading towards point A both with a speed of 50 km/hr meets each other at 01.00 P.M ,

a) **What are the time taken by moped1 and moped2 to cover their respective remaining distances?**
b) **What is the distance between point A and B?**

Solution:

a) Here, a = 4 hours.

$6 = t_1+4$ (From equation 22)

Hence t_1 = 2 hours

t_2 = t = 6 hours (From equation 23)

Hence moped1 and moped 2 take 2 and 6 hours respectively to cover their remaining distances after their meeting point.

b) d = 50*((2*6)-4) = 400 km (From equation 17).

Hence distance between point A and point B is 400 km.

Case d): When 2 vehicles start at same time with same speed from points A and B and move towards each other:

In this case, d-m = m and $S_1 = S_2$

$$m = d/2 \quad \text{(24)}$$

$$2*S_1*t = d \quad \text{(25)}$$

$$t_1 = t_2 = t \quad \text{(26)}$$

Example: When a auto1 starting from point A heading towards point B and auto2 starting from point B heading towards point A both travelling at a speed of 60 km/hr meet each other after 6 hours,

a) **What are the time taken by auto 1 and 2 to cover their remaining distances if they start their journey simultaneously?**
b) **What is the distance between point A and B?**
c) **What is the distance from A to the meeting point?**

Solution:

a) $t_1 = t_2 = t = 6$ hours (From equation 26)

Hence auto1 and auto 2 both take 6hours to cover their remaining distances after their meeting point.

b) d = 2*60*6 = 720 km (From equation 25)

Hence distance between point A and point B is 720 km.

c) m = 720/2 =360 km (From equation 24)

Hence distance from point A to the meeting point is 360 km.